END OF SEMESTER EXAMINATIOON

COURSE TITLE: CALCULUS

COURSE CODE: EBS 301

SECTION A

1. Find the gradient of the normal to the curve $x^2 - 2\text{xy} + 3y^2 - \text{x} = 4$ at the point (2,-1).

A. 2

B. 1

C. $\frac{1}{2}$

D. $\frac{-1}{2}$

2. Given that $x = 3r^2 - 2\text{r}$ and $y = 2r^2 + 2$, find $\frac{dy}{dx}$ in terms of r.

A. $24r^2 - 8\text{r}$

B. $\frac{2r}{3r-1}$

C. $24r^2 + 8\text{r}$

D. $\frac{2r-1}{3r^2-r}$

3. An ink blots on a piece of paper and spreads at the rate of $0.5cm^2/\text{s}$. Find the rate at which the radius of the circular blot is increasing when the radius is 1cm.

A. $\frac{1}{2\pi}\text{cm/s}$

B. $\frac{1}{3\pi}\text{cm/s}$

C. $\frac{2}{3\pi}\text{cm/s}$

D. $\frac{1}{4\pi}\text{cm/s}$

4. Evaluate $\int_2^5 (2x - 5x^4)dx$

   A. -3072

   B. -3115

   C. -3100

   D. -3037

5. Find the integral of the function $f: x \to 3x^2 - 5$ with respect to *x*.

   A. $x^3 - 5x + c$

   B. $x^3 + 5x + c$

   C. $x^3 - 5 + c$

   D. $x^3 - 5x^2 + c$

Section B

1. a) A gardener has 200m of metal railing with which to form two adjacent sides of a rectangular enclosure, the other two sides being two existing walls of the yard, meeting at right angle. Calculate the greatest possible area of the enclosure.

   b) Find the gradient of the ellipse $x^2 - 3xy + 2y^2 - 2x = 4$ at the point $(1, -1)$

   c) Evaluate $\int_1^4 \frac{(2x+4)}{x^3} dx$

2. a) Sketch curve

$y = x^3 - 9x^2 + 15x - 7$

b) Kofi was asked to measure the surface area of a sphere. In the process, he committed an error of 2% in the measurement of the radius. Calculate the corresponding percentage error he is likely to commit in the measurement of the surface area.

c. A tricycle rider starts from rest and covers a distance of S(m) in t(s), where $S = \frac{1}{6}t^3 + \frac{1}{4}t^2$. Calculate,

i. the initial acceleration of the rider

ii. the acceleration of the rider at the end of the 2nd second.

Answers to section A

1. **D**
2. **B**
3. **D**
4. **A**
5. **A**

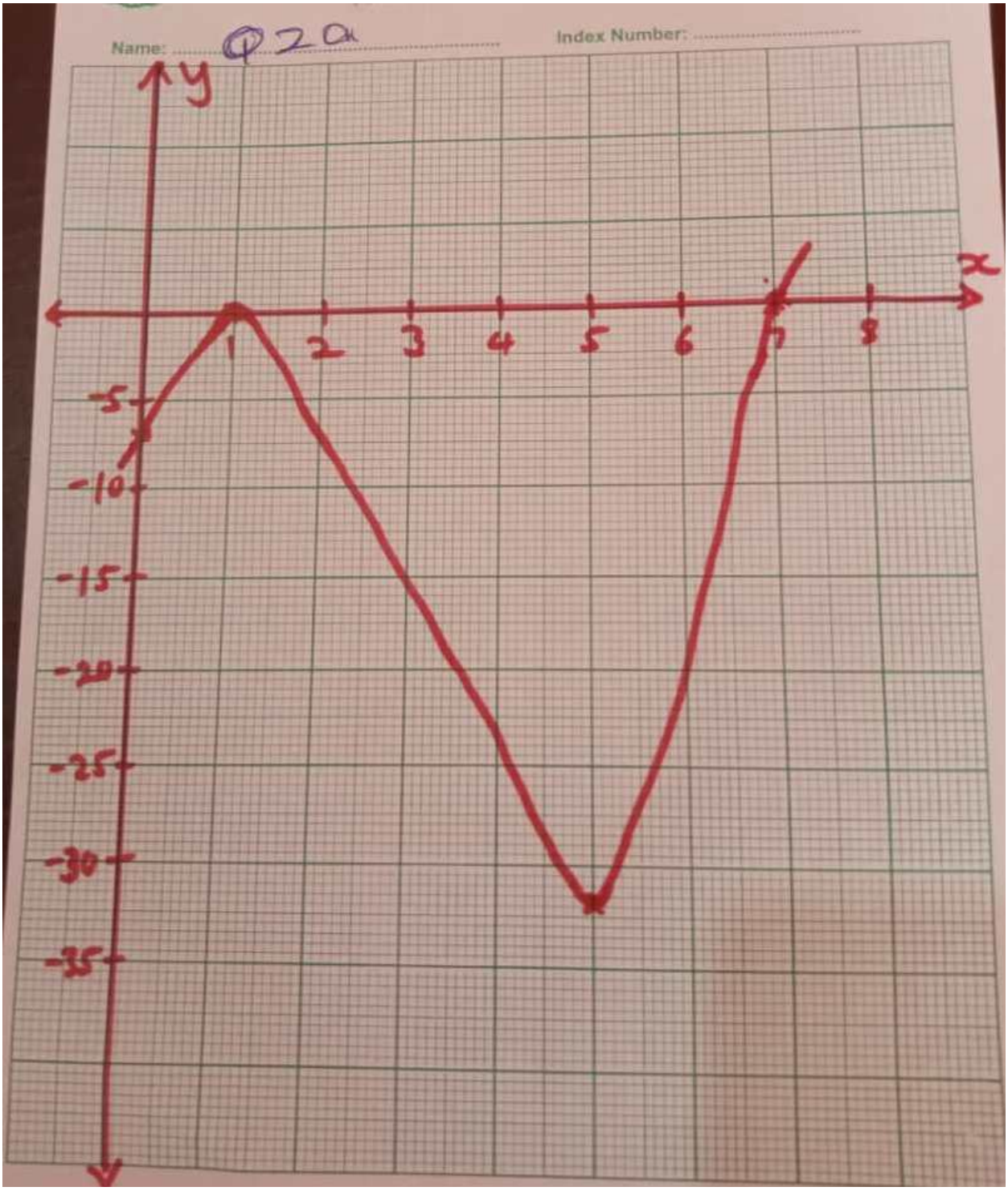
RUBRICS FOR SECTION B

Question Number	Details	Marks
1.(a)	$x + y = 200$ or its equivalence	M1 For correct equation
	$x = 200 - y$	
	$A = xy$…….. *	
	$A = (200 - y)y$ or its equivalence	M1 for correct substitution into *
	$\frac{dA}{dy} = 200 - 2y$ or its equivalence	M1 for $\frac{dA}{dy}$
	$200 = 2y$	M1 for simplifying
	$y = 100m, \quad x = 100m$	A1 for either $y = 100$ or $x = 100$
	$A = 100 \times 100$	
	$A = 10000m^2$	M1A1 for $A = 10000m^2$ $(-1OU/WU, once\ only)$
b)	$2x - 3\left(y + x\frac{dy}{dx}\right) + 4y\frac{dy}{dx} - 2 = 0$	M1
	$(4y - 3x)\frac{dy}{dx} = 2 - 2x + 3y$	M1 for solving
	$\frac{dy}{dx} = \frac{2 + 3y - 2x}{4y - 3x}$	M1A1
	$\frac{dy}{dx} = \frac{2 + 3(-1) - 2(1)}{4(-1) - 3(1)}$	M1 for correct substitution
	$\frac{dy}{dx} = \frac{3}{7}$	A1 for answer

c)	$\int_1^4 \frac{2x+4}{x^3} = \left[\frac{2x^{-2+1}}{-2+1} + \frac{4x^{-3+1}}{-3+1}\right]$ $= \left[\frac{-2}{x} - \frac{2}{x^2}\right]_1^4$ $= \left[\frac{-2}{4} - \frac{2}{4^2}\right] - [-2-2]$ $= \frac{-4-1}{8} + 4$ $= \frac{-5+32}{8} = \frac{27}{8}$	**M1** **M1A1 for simplifying to obtain** $\left[\frac{-2}{x} - \frac{2}{x^2}\right]_1^4$ **M1 for correct substitution** **M1 for solving** **M1A1 for simplifying to obtain** $\frac{27}{8}$ **[20 marks]**
2.a)	**y-intercept =** $(0,-7)$ **x-intercept =** $(7,0),(1,0)$ **Turning points** $\frac{dy}{dx} = 3x^2 - 18x + 15$ $3x^2 - 18x + 15 = 0$	**B1 for y-intercept or x-intercepts** **M1 for** $\tan^{-1}\left(\frac{4}{3}\right) = 53.13$

2.a)	$x = 1 \text{ or } 5$ $(1, 0)$ max. pt $(5, -32)$ min. pt See attached for graph	M1A1 solving to obtain the turning values A1 for obtaining the max. pt A1 for obtaining the min. pt B4 for graph. (-1ee)
2.b)	$\frac{dS}{dr} = 8\pi r$ $\delta S = 8\pi r \times 0.02r = \frac{4\pi r^2}{25}$ $Percentage\ error\ in\ area = \frac{\delta S}{S} \times 100\%$ $= \frac{4\pi r^2}{25} \div 4\pi r^2 \times 100\%$ $= \frac{4\pi r^2}{25} \times \frac{1}{4\pi r^2} \times 100\%$ $= 0.04 \times 100$ $= 100\%$	M1 A1 M1 M1A1
c)	$S = \frac{1}{6}t^3 + \frac{1}{4}t^2$ $\frac{dS}{dt} = v = \frac{1}{2}t^2 + \frac{1}{2}t$ $\frac{d^2S}{dt^2} = a = t + \frac{1}{2}$	M1 for $\frac{dS}{dt} = a = \frac{1}{2}t^2 + \frac{1}{2}t$ M1
i)	When $t = 0$,	

ii)	$a = 0 + \frac{1}{2} = \frac{1}{2} m/s^2$ **When $t = 2$** $a = 2 + \frac{1}{2} = 2\frac{1}{2} m/s^2$	**A1** **M1A1** **[20 marks]**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